DESIGN METHODOLOGY FOR FUZZY LOGIC CONTROLLERS

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ABSTRACT

The paper presents an insight into the mathematical basis of fuzzy linguistic modeling and discusses the sensitivity of fuzzy rule-based systems with respect to membership function parameters, rule structure, and inference mechanisms. Examples with different inference models are analyzed and compared in order to illustrate their influence on the behavior and performance of fuzzy systems. Particular attention is devoted to the effects of parameter variations on system stability, robustness, accuracy, and interpretability. The study highlights the importance of sensitivity analysis in the design of reliable fuzzy controllers.

Keywords: Fuzzy logic, linguistic variable, inference, controller.

INTRODUCTION

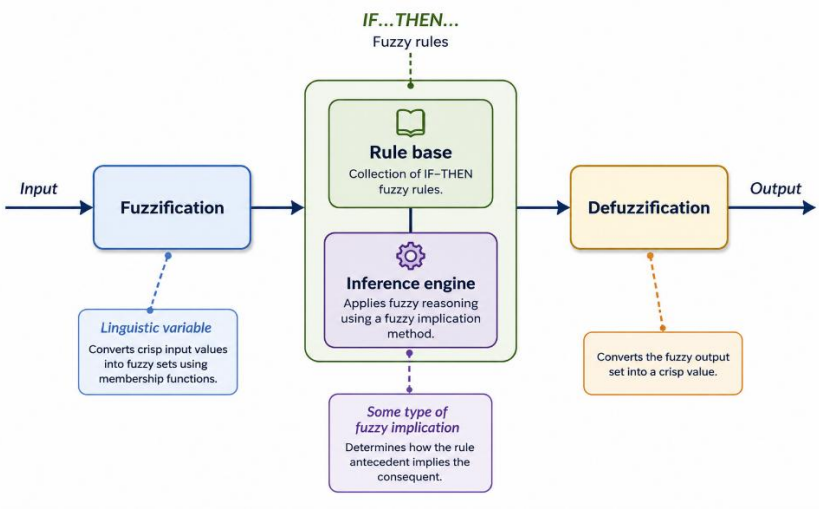
Fuzzy sets were introduced by Lotfi Zadeh in 1965 as a way to overcome the imprecision and uncertainty that arise in certain aspects and domains of human reasoning [1]. The source of imprecision is considered to be the lack of clear criteria, as well as the subjectivity and uncertainty present in many problems. Such problems frequently occur when dealing with classes of objects in the real world, which is why fuzzy sets have found applications in many fields such as psychology, sociology, economics, and other scientific disciplines [2], [3], [4], [1].

Classical two-valued (bivalent) logic is a formal mathematical discipline focused on examining the principles and techniques of reasoning, where new conclusions are generated from established propositions. In classical logic, it is assumed that every proposition has exactly one of two truth values: true or false, typically denoted by 1 for true and 0 for false. Fuzzy logic serves as a natural expansion and generalization of traditional logic, permitting propositions to hold a degree of truth instead of being strictly true or false. As a result, the truth value of a proposition is no longer limited to the binary set $\{0,1\}$, but can assume any value from a wider range, typically the real interval $[0,1]$. In this way, fuzzy logic presents the idea of graded truth, thus offering a mathematical structure for depicting the vagueness and uncertainty intrinsic to human reasoning and natural language. This generalization facilitates approximate reasoning, which involves

generating graded conclusions (fuzzy propositions) based on a set of vague or partially accurate premises. In contrast to traditional logical systems that demand exactness, fuzzy logic is intentionally formulated to represent reasoning in contexts marked by vagueness, partial information, and unpredictability.

The main goal of fuzzy logic is to create a strong theoretical base for approximate reasoning with vague statements, using fuzzy set theory as its core mathematical instrument and key analytical resource. A fuzzy controller is a control system that uses a collection of linguistic IF-THEN rules to regulate a system's behavior based on the current values of its input variables. Fuzzy systems, including fuzzy controllers, were first practically applied in 1975, when Ebrahim Mamdani developed a fuzzy controller for controlling a steam engine. His work demonstrated that complex control tasks could be successfully performed using linguistic IF–THEN rules instead of precise mathematical models. This achievement marked the beginning of fuzzy control as a practical engineering discipline and inspired numerous applications in industrial automation, robotics, automotive systems, and consumer electronics [5].

Fuzzy logic, rather than exact mathematical functions, is used to describe the relationships between variables. Fuzzification, a rule base, an inference mechanism, and defuzzification are the four primary parts of a fuzzy control system. Fuzzy sets with linguistic labels like “low,” “medium,” or “high” are created from crisp numerical inputs during the fuzzification stage. In the form of IF-THEN rules that specify the control strategy, the rule base comprises expert knowledge. These rules are assessed and combined by the inference mechanism to provide a fuzzy output. The combined fuzzy output is finally transformed into a single clear control action that can be implemented during the defuzzification stage. Also, fuzzy logic follows an approach that resembles human reasoning and is relatively simple to understand. It has been widely applied in various fields and has proven effective in solving numerous problems. Based on set theory, fuzzy logic employs fuzzy models and algorithms to reduce the differences between formal and natural languages. Nevertheless, fuzzy logic also has some limitations, including the lack of precisely defined rules, possible imprecision in reasoning, the necessity of extensive hardware testing for fuzzy systems, and longer data processing times in comparison with some classical approaches [6], [7], [4], [8], [9].



Picture 1. Parts of fuzzy control system.

BASIC CONCEPTS, FUZZY IF-THEN RULES

First, we will give an overview of the basic concepts and results of the fuzzy logic theory. Let X be the set we call the universe. The fuzzy set A is defined as a set of ordered pairs

$A = \{(x, \mu_A(x)) | x \in X\}$,

where

$\mu_A: X \to [0,1]$

is a membership function that assigns each element of the universe X some value from the interval $[0,1]$.

Various types of membership function can be used to describe these fuzzy sets: linear, triangular, trapezoidal, bell and Gaussian functions. A fuzzy set A on the discrete universe $X = \{x_1, x_2, \ldots, x_n\}$ can be represented as

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \cdots + \frac{\mu_A(x_n)}{x_n},$$

that is

$$A = \sum_{x \in X} \frac{\mu_A(x)}{x}.$$

The intersection of the fuzzy sets A and B is the fuzzy set C whose membership function is given by

$\mu_C(x) = \mu_A(x) \wedge \mu_B(x), x \in X.$

The union of fuzzy sets A and B is a fuzzy set C whose membership function is given by

$\mu_C(x) = \mu_A(x) \vee \mu_B(x), x \in X.$

The complement of the fuzzy set A is the fuzzy set A' whose membership function is given by

$\mu_{A'}(x) = 1 - \mu_A(x), x \in X.$

A binary fuzzy relation, denoted by $\mathcal{R}$, is a fuzzy set defined on the Cartesian product of non-empty sets $X_1 \times X_2$. The mapping $\mu_R: X_1 \times X_2 \to [0,1]$, where $\mu_R(x_1, x_2)$ represents the degree to which the element (x_1, x_2) belongs to the relation $\mathcal{R}$ is called the membership function of the set $\mathcal{R}$

$\mathcal{R} = \{((x_1, x_2), \mu_R(x_1, x_2)) | (x_1, x_2) \in X_1 \times X_2, \mu_R(x_1, x_2) \in [0,1]\}.$

Fuzzy relations have many uses and are an extension of classical relations. In contrast to classical relations, which show whether or not things are related, fuzzy relations let elements to be somewhat related.

A fuzzy IF-THEN rule is a rule of the form: IF p THEN q, where p and q are corresponding propositions. In classical logic, the following property holds

$p \Rightarrow q \equiv \neg p \vee q.$

In fuzzy logic, the expression IF p THEN q is represented by a membership function $\mu: U_1 \times U_2 \to [0,1]$ defined by

$\mu(p, q) = \neg p \vee q,$

for $p \in U_1, q \in U_2$. Depending on which operator is used to model the complement and which operator is used to model the fuzzy union, various different fuzzy implications are obtained

$\mu_Z(x, y) = \max(\min(\mu_{p1}(x), \mu_{p2}(y)), 1 - u_{p1}(x))$ – Zadeh implication,

$\mu_L(x, y) = \min(1, 1 - \mu_{p1}(x) + \mu_{p2}(y))$ – Lukasiewicz implication,

$\mu_D(x, y) = \max(1 - \mu_{p1}(x), \mu_{p2}(y))$ – Dienes-Rescher implication,

$\mu_{MP}(x, y) = (\mu_{p1}(x)\mu_{p2}(y))$ – Mamdani 2 implication.

We extend the fundamental rules of inference from classical logic. **Modus ponens**: If p is true and $p \Rightarrow q$ is true, then q is also true, i.e.

$(p \wedge (p \Rightarrow q)) \Rightarrow q.$

Intuitively, modus ponens means

Premise 1: x is equal A;

Premise 2: IF x is equal A, THEN y is equal B;

Conclusion: y is equal B.

Generalization of modus ponens: If the proposition "x is equal to A'" holds, and the proposition "if x is equal to A, then y is equal to B" holds, then the proposition "y is equal to B'" also holds, in the sense that the closer A' is to A, the closer B' is to B.

Generalization of modus ponens means

Premise 1: x is equal A';

Premise 2: IF x is equal A, THEN y is equal B;

Conclusion: y is equal B'.

For given fuzzy set $A': U \to [0,1]$, which represents premise "x is equal A'", and fuzzy relation $A \to B \in \mathcal{R}(U,V)$, which represents premise "IF x is equal A, THEN y is equal B", fuzzy set $B': V \to [0,1]$, which represents conclusion "y is equal B'", we represent as

$$\mu_{B'} = \sup_{x \in U}(\mu_{A'}(x), \mu_{A \to B}(x,y)).$$

Triangular norm (t-norm) is a binary operation used in fuzzy logic and fuzzy set theory to model the intersection and union of fuzzy sets. T-norm is function $T: [0,1]^2 \to [0,1]$ that for $a, b, c \in [0,1]$ is valid

boundary condition $T(a,1) = a$;

commutativity $T(a,b) = T(b,a)$;

monotonicity, if $b_1 \leq b_2$ then $T(a,b_1) \leq T(a,b_2)$;

associativity $T(a,T(b,c)) = T(T(a,b),c)$.

EXAMPLE

Consider the sets $U = \{x_1, x_2, x_3\}$ and $V = \{y_1, y_2\}$. The fuzzy sets are given as follows:

$$A = \frac{0.5}{x_1} + \frac{1}{x_2} + \frac{0.6}{x_3}, \quad A' = \frac{0.6}{x_1} + \frac{0.9}{x_2} + \frac{0.7}{x_3}, \quad B = \frac{1}{y_1} + \frac{0.4}{y_2}.$$

Using the generalized Modus ponens rule, determine B assuming that the t-norm is the minimum, and that the implication representing the statement $A \to B$ is given as follows

Zadeh implication;

1. Lukasiewicz implication;
2. Dienes-Rescher implication;
3. Mamdani 2 implication.

SOLUTION

Since the generalized Modus ponens rule is applied, the membership function is defined as follows

$$B'(y) = \sup_{x \in U} t[\mu_{A'}(x), \mu_{A \to B}(x,y)],$$

t-norm is minimum, then we have

$$B'(y) = \sup_{x \in U} \min[\mu_{A'}(x), \mu_{A \to B}(x,y)].$$

1. Using Zadeh's implication, we have

$$B'(y) = \sup_{x \in U} \min[\mu_{A'}(x), \max[\min[A(x), A(y)], 1 - A(x)]].$$

- For $y = y_1$ is valid

$$B'(y_1) = \sup_{x \in U} \min[\mu_{A'}(x), \max[\min[A(x), \mu_B(y_1)], 1 - A(x)]]$$

$$= \sup \begin{cases} \min[0.6, \max[\min[0.5,1], 1 - 0.5]], & x = x_1, \\ \min[0.9, \max[\min[1,1], 1 - 1]], & x = x_2, \\ \min[0.7, \max[\min[0.6,1], 1 - 0.6]], & x = x_3 \end{cases}$$

$$= \sup \begin{cases} 0.5, & x = x_1, \\ 0.9, & x = x_2, \\ 0.6, & x = x_3 \end{cases}$$

$$= 0.9.$$

- For $y = y_2$ is valid

$$B'(y_2) = \sup_{x \in U} \min[\mu_{A'}(x), \max[\min[A(x), \mu_B(y_2)], 1 - A(x)]]$$

$$= \sup \begin{cases} \min[0.6, \max[\min[0.5, 0.4], 1 - 0.5]], & x = x_1, \\ \min[0.9, \max[\min[1, 0.4], 1 - 1]], & x = x_2, \\ \min[0.7, \max[\min[0.6, 0.4], 1 - 0.6]], & x = x_3 \end{cases}$$

$$= \sup \begin{cases} 0.5, & x = x_1, \\ 0.4, & x = x_2, \\ 0.4, & x = x_3 \end{cases}$$

$$= 0.5.$$

- Finally, we have that $B'(y_1) = 0.9$ and $B'(y_2) = 0.5$.

2. Using the Lukasiewicz implication, we have

$$B'(y) = \sup_{x \in U} \min[A'(x), \min[1, 1 - A(x) + B(y)]].$$

- For $y = y_1$ is valid

$$B'(y_1) = \sup_{x \in U} \min[A'(x), \min[1, 1 - A(x) + B(y_1)]]$$

$$= \sup \begin{cases} \min[0.6, \min[1, 1 - 0.5 + 1]], & x = x_1, \\ \min[0.9, \min[1, 1 - 1 + 1]], & x = x_2, \\ \min[0.7, \min[1, 1 - 0.6 + 1]], & x = x_3 \end{cases}$$

$$= \sup \begin{cases} 0.6, & x = x_1, \\ 0.9, & x = x_2, \\ 0.7, & x = x_3 \end{cases}$$

$$= 0.9.$$

- For $y = y_2$ is valid

$$B'(y_2) = \sup_{x \in U} \min[A'(x), \min[1, 1 - A(x) + B(y_2)]]$$

$$= \sup \begin{cases} \min[0.6, \min[1, 1 - 0.5 + 0.4]], & x = x_1, \\ \min[0.9, \min[1, 1 - 1 + 0.4]], & x = x_2, \\ \min[0.7, \min[1, 1 - 0.6 + 0.4]], & x = x_3 \end{cases}$$

$$= \sup \begin{cases} 0.6, & x = x_1, \\ 0.4, & x = x_2, \\ 0.7, & x = x_3 \end{cases}$$

$$= 0.7.$$

- Finally, we have that $B'(y_1) = 0.9$ and $B'(y_2) = 0.7$.

3. Using Dienes-Rescher implication, we have

$$B'(y) = \sup_{x \in U} \min[\mu_{A'}(x), \max[1 - A(x), B(y)]].$$

- For $y = y_1$ is valid

$$B'(y_1) = \sup_{x \in U} \min[\mu_{A'}(x), \max[1 - A(x), B(y_1)]]$$

$$= \sup \begin{cases} \min[0.6, \max[1 - 0.5, 1]], & x = x_1, \\ \min[0.9, \max[1 - 1, 1]], & x = x_2, \\ \min[0.7, \max[1 - 0.6, 1]], & x = x_3 \end{cases}$$

$$= \sup \begin{cases} 0.6, & x = x_1, \\ 0.9, & x = x_2, \\ 0.7, & x = x_3 \end{cases}$$

$$= 0.9.$$

- For $y = y_2$ is valid

$$B'(y_2) = \sup_{x \in U} \min[\mu_{A'}(x), \max[1 - A(x), B(y_2)]]$$
$$= \sup \begin{cases} \min[0.6, \max[1 - 0.5, 0.4]], & x = x_1, \\ \min[0.9, \max[1 - 1, 0.4]], & x = x_2, \\ \min[0.7, \max[1 - 0.6, 0.4]], & x = x_3 \end{cases}$$
$$= \sup \begin{cases} 0.5, & x = x_1, \\ 0.4, & x = x_2, \\ 0.4, & x = x_3 \end{cases}$$
$$= 0.5.$$

- Finally, we have that $B'(y_1) = 0.9$ and $B'(y_2) = 0.5$.

4. Using the Mamdani 2 implication, we have

$$B'(y) = \sup_{x \in U} \min[\mu_{A'}(x), A(x)B(y)].$$

- For $y = y_1$ is valid

$$B'(y_1) = \sup_{x \in U} \min[\mu_{A'}(x), A(x)B(y_1)]$$
$$= \sup \begin{cases} \min[0.6, 0.5 \cdot 1], & x = x_1, \\ \min[0.9, 1 \cdot 1], & x = x_2, \\ \min[0.7, 0.6 \cdot 1], & x = x_3 \end{cases}$$
$$= \sup \begin{cases} 0.5, & x = x_1, \\ 0.9, & x = x_2, \\ 0.6, & x = x_3 \end{cases}$$
$$= 0.9.$$

- For $y = y_2$ is valid

$$B'(y_2) = \sup_{x \in U} \min[\mu_{A'}(x), A(x)B(y_2)]$$
$$= \sup \begin{cases} \min[0.6, 0.5 \cdot 0.4], & x = x_1, \\ \min[0.9, 1 \cdot 0.4], & x = x_2, \\ \min[0.7, 0.6 \cdot 0.4], & x = x_3 \end{cases}$$
$$= \sup \begin{cases} 0.2, & x = x_1, \\ 0.4, & x = x_2, \\ 0.24, & x = x_3 \end{cases}$$
$$= 0.4.$$

- Finally, we have that $B'(y_1) = 0.9$ and $B'(y_2) = 0.4$.

CONCLUSION

The presented example demonstrates that the generalized modus ponens can be successfully applied using different fuzzy implication operators. Although the same fuzzy sets and the same inference procedure were used in all cases, the resulting fuzzy conclusions were not identical. This confirms that the selected implication operator has a significant influence on the outcome of the inference process. The comparison also shows that different implication models preserve and propagate uncertainty in different ways. Therefore, the choice of a fuzzy implication should be made according to the requirements of the specific application, since it directly affects the behavior, accuracy, and interpretability of the resulting fuzzy controller.

The proposed methodology establishes a systematic approach to the design and implementation of fuzzy controllers for systems involving uncertainty and imprecise information. Their capability to represent human expertise through linguistic rules, together with their flexibility and transparency, makes fuzzy controllers a valuable solution for a wide range of control applications. Future research may focus on improving controller performance through the optimization of membership functions, the automatic generation of fuzzy rules, and the combination of fuzzy logic with adaptive and machine learning methods.

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DECLARATIONS OF INTEREST STATEMENT

The authors affirm that there are no conflicts of interest to declare in relation to the research presented in this paper.

LITERATURE

1. Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.
2. Dernoncourt, F. Introduction to Fuzzy Logic; Massachusetts Institute of Technology: Cambridge, MA, USA, 2013. *Availiable Online: http://aisii. azc. uam. mx/mcbc/Cursos/IntCompt/Lectura15. pdf (accessed on 16 October 2021)*.
3. Hájek, P. (2001). *Metamathematics of fuzzy logic* (Vol. 4). Springer Science & Business Media.
4. Novák, V. (2005). On fuzzy type theory. *Fuzzy Sets and Systems*, *149*(2), 235-273.
5. Mamdani, E., Assilian, S. (1974). *Case Study on the Application of Fuzzy Set Theory to Automatic Control*. In the Proceedings of IFAC Stochastic Control Symposium, Budapest.
6. Belohlavek, R. (2012). *Fuzzy relational systems: foundations and principles*. Springer Science & Business Media.
7. Bloom, N. (2014). Fluctuations in uncertainty. *Journal of economic Perspectives*, *28*(2), 153-176.
8. Novák, V., Perfilieva, I., & Dvorak, A. (2016). *Insight into fuzzy modeling*. John Wiley & Sons.
9. Zimmermann, H. J. (2011). *Fuzzy set theory—and its applications*. Springer Science & Business Media.
10. Dubois, D., Esteva, F., Godo, L., Prade, H., *Fuzzy-set based logics an history oriented presentation of their main developments*, 2007.
11. Klement, E., Mesiar, R., Pap, E., *Triangular Norms*, 2013.
12. Micić, I., *Fazi skupovi, fazi logika i primene*, Zbirka zadataka, 2019.
13. Pedryc, W., *Fuzzy systems engineering: Toward Human-centric Computing*, 2007.
14. Takagi, T., Sugeno, M., *Fuzzy identification of systems and its applications to modeling and control*, 1985.
15. Zadeh, L., *Fuzzy logic, neural networks and soft computing*, Communications of the ACM, 1994.