

COMPARISON OF MULTI-CRITERIA DECISION-MAKING METHODS: IS IT ALL THE SAME?

Nenad Medić, Zoran Anišić

University of Novi Sad, Faculty of Technical Sciences, Trg Dositeja Obradovića 6, 21 000 Novi Sad, Serbia, anisic@uns.ac.rs

ABSTRACT

Multi-criteria decision making (MCDM) methods have been used in various research areas in order to evaluate and rank finite number of alternatives based on a set of predetermined criteria. One of the main concerns when applying MCDM methods is related to the fact that different methods can yield different results in terms of the ranking of the proposed alternatives. This is usually explained with the subjectivity in the decision-making process which is incorporated in the most of the MCDM methods. The aim of this paper is to show that different methods can yield different results even when the subjectivity is reduced to the minimum. For this purpose, five of the most used MCDM methods (i.e., PROMETHEE, ELECTRE, AHP, TOPSIS, and VIKOR) were analysed and compared on a general example. As expected, the ranking of the alternatives is not the same for all of the used methods. Based on the results presented in this paper, researchers and practitioners should be motivated to investigate more thoroughly the specificities of various MCDM methods and consequently make more reliable decisions based on the use of appropriate methods for their specific problem.

Keywords: Decision-making, MCDM, Comparative analysis.

INTRODUCTION

Decision making is extremely intuitive when considering problems based on only one criterion, because the only task is to choose the alternative with the highest preference according to the given criterion. However, when a decision maker needs to make a decision based on a number of criteria, various problems arise. Some of the issues that should be considered are defining the importance of the criteria, the dependence between the elements, and various conflicts among the criteria. To overcome these and many other problems, the application of sophisticated decision-making methods is required (Hwang, & Yoon, 1981). In order to solve the problem of multi-criteria decision making (MCDM), there is a finite number of steps that should be implemented. Initially, decision maker needs to identify the problem and define the attributes and criteria to solve the problem appropriately. Furthermore, it is necessary to collect data and information which will enable the decision maker to clearly express his preferences. After that, it is necessary to identify a set of possible alternatives or strategies that enable the realization of the goal. Finally, it is important to identify appropriate methods that will provide an adequate choice among alternatives (Opricovic, & Tzeng, 2004). Identification and selection of appropriate method for solving the defined problem is considered as one of the major concerns in the research field since different methods can yield different results in terms of the ranking of the proposed alternatives. The problem of selecting appropriate MCDM method has been addressed in literature (Guitouni, & Martel, 1998; MacCrimmon, 1972; Ozernoy, 1992). However, there is no perfect solution for this problem, because of the differences in the approach based on the method used. The reasons for the inconsistencies in results are numerous. The literature recognizes following reasons as the most significant: algorithm for selection of preferred alternative differs from method to method, different methods use criteria weights differently in their computations, algorithms include scaling of the objectives, which can affect the weights already chosen, and some methods introduce

additional parameters that affect selection of preferred alternative (Zanakis, Solomon, Wishart, & Dublisch, 1998).

All of these issues opened a debate among researchers in order to find similarities and differences between various MCDM methods which has been used previously in the literature. For example, through comparison of VIKOR, TOPSIS and SAW authors identified that decision made with traditional method such as SAW is not clear enough compared to decision obtained with TOPSIS or VIKOR (Chu, Shyu, Tzeng, & Khosla, 2007). In another seminal work, comparison of VIKOR and TOPSIS revealed that different approaches in normalization of attributes and aggregation functions affect the order of alternatives (Opricovic, & Tzeng, 2004). In (Opricovic & Tzeng, 2007), VIKOR was compared with TOPSIS, ELECTRE, and PROMETHEE in order to recognize differences and similarities between methods.

This paper aims to compare results obtained using PROMETHEE, ELECTRE, AHP, TOPSIS, and VIKOR on an empirical example. These MCDM methods were analyzed and compared on a real-world decision-making problem (i.e. cookies recipe selection). One of the reasons to compare these methods is found in the fact that they are the most used MCDM methods for various types of problems (Mardani et al., 2015). Furthermore, these methods have different approach to problem solving. PROMETHEE, ELECTRE, and AHP are outranking methods, while TOPSIS and VIKOR are compromise ranking methods. The contribution of this paper to the existing literature is reflected through confirmation on an empirical example that there are differences in the order of preferred alternatives based on the selected approach for problem solving.

The rest of the paper is structured as follows. In Section 2 all of the methods used (i.e., PROMETHEE, ELECTRE, AHP, TOPSIS, and VIKOR) are explained. In Section 3 the methods are applied on an empirical example and results followed by discussion are presented. Finally, Section 4 presents conclusion of the paper including limitations of the study and propositions for further research.

METHODS

In this section PROMETHEE, ELECTRE, AHP, TOPSIS, and VIKOR were described as a theoretical basis for the following application.

The PROMETHEE family of outranking methods, including the PROMETHEE I for partial ranking of the alternatives and the PROMETHEE II for complete ranking of the alternatives, were developed by Brans (Brans, 1982). PROMETHEE II is described in this part of the paper, since majority of researchers have referred to this version of the method (Behzadian Khanmohammadi, Yazdani, & Ignatius, 2010). This method is based on a pairwise comparison of alternatives in respect to each defined criterion. The implementation of PROMETHEE II requires two types of information. Decision maker needs to define weight and preference function for each criterion. Weight determines the importance of each criterion, while preference function serves to translate difference between the evaluations obtained by alternatives into a preference degree ranging from zero to one. There are six types of preference functions proposed: (1) usual criterion, (2) U-shape criterion, (3) V-shape criterion, (4) level criterion, (5) V-shape with indifference criterion and (6) Gaussian criterion (Vincke, 1985). The procedure of PROMETHEE II method is as follows (Brans, 1982):

Step 1: Determination of deviations based on pairwise comparisons

$$d_j(a, b) = g_j(a) - g_j(b) \quad (1)$$

Where $d_j(a, b)$ denotes the difference between the evaluations of a and b on each criterion.

Step 2: Application of the preference function

$$P_j(a, b) = F_j[d_j(a, b)] \quad j = 1, \dots, k \quad (2)$$

Where $P_j(a, b)$ denotes the preference of alternative a with regard the alternative b on each criterion, as a function of $d_j(a, b)$.

Step 3: Calculation of an overall or global preference index

$$\forall a, b \in A, \quad \pi(a, b) = \sum_{j=1}^k P_j(a, b) w_j \quad (3)$$

Where $\pi(a, b)$ of a over b (from 0 to 1) is defined as a weighted sum $p(a, b)$ of each criterion, and w_j is the weight associated with the expressing the decision maker's preference as the relative importance of the j -th criterion.

Step 4: Calculation of outranking flows

$$\phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \pi(a, x) \quad (4)$$

$$\phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \pi(x, a) \quad (5)$$

Where $\phi^+(a)$ and $\phi^-(a)$ denote the positive outranking flow and negative outranking flow for each alternative, respectively.

Step 5: Calculation of net outranking flow

$$\phi(a) = \phi^+(a) - \phi^-(a) \quad (6)$$

Step 6: Determine the ranking of all the considered alternatives depending on the values of $\phi(a)$. Higher value of $\phi(a)$, means better ranking of the alternative. Thus, the best alternative is the one having the highest $\phi(a)$ value.

The ELECTRE method was originally developed by (Benayoun, Roy, & Sussman, 1966). The original method derived into ELECTRE I and the evolutions have subsequently continued with ELECTRE II (Roy, & Bertier, 1973), ELECTRE III (Roy, 1978), ELECTRE IV, and ELECTRE TRI (Yu, 1992). ELECTRE was the first method to include an outranking approach based on pairwise comparisons between alternatives under each criterion separately. The procedure of ELECTRE method is as follows (Anojkumar, Ilangkumaran, & Sasirekha, 2014):

Step 1: Normalization of the evaluation matrix

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^m x_{ij}^2}} \quad (7)$$

Step 2: Construction of the weighted normalized decision matrix

$$v_{ij} = w_i * r_{ij} \quad (8)$$

Step 3: Ascertainment of Concordance and Discordance interval sets

$$C_{ab} = \{j | x_{aj} \geq x_{bj}\} \quad (9)$$

$$D_{ab} = \{j | x_{aj} \leq x_{bj}\} = j - C_{ab} \quad (10)$$

Step 4: Computation of concordance interval matrix

$$C_{ab} = \sum_{j \in C_{ab}} W_j \quad (11)$$

Step 5: Computation of discordance interval matrix

$$d(a, b) = \frac{\max_{j \in D_{ab}} |v_{aj} - v_{bj}|}{\max_{j \in J, m, n \in I} |v_{mj} - v_{nj}|} \quad (12)$$

Step 6: Computation of concordance interval matrix

$$\bar{c} = \sum_{a=1}^m \sum_b^m \frac{c(a,b)}{m(m-1)} \quad (13)$$

Step 7: Determine the discordance index matrix

$$\bar{d} = \sum_{a=1}^m \sum_b^m \frac{d(a,b)}{m(m-1)} \quad (14)$$

Step 8: Computation of net superior and inferior value

$$c_a = \sum_{b=1}^n c(a,b) - \sum_{b=1}^n c(b,a) \quad (15)$$

$$d_a = \sum_{b=1}^n d(a,b) - \sum_{b=1}^n d(b,a) \quad (16)$$

The AHP method was developed by Saaty (Saaty, 1980). It is based on pairwise comparison using a nine-point scale. AHP is one of the most commonly used techniques for solving complex MCDM problems. The procedure of AHP is as follows (Saaty, 1980):

Step 1: Define the decision problem and determine its goal.

Step 2: Establish the decision hierarchy in which the highest level denotes the goal of the decision problem, the middle level represents multiple criteria, and the lowest level indicates the alternatives.

Step 3: Compare the elements in the corresponding level in pairs according to their levels of influence on the specified element in the higher level of the decision hierarchy.

Step 4: Synthesize all of the pairwise comparison matrices to find the relative priority for each criterion/alternative

Step 5: Check the consistency of the pairwise comparison judgement. In order to calculate matrix Consistency Ratio (CR), first the matrix Consistency Index (CI) is calculated:

$$CI = (\lambda_{max} - n)/(n - 1) \quad (17)$$

where λ_{max} is the largest eigenvalue and n is the matrix order. After that, CR is calculated:

$$CR = CI/RCI \quad (18)$$

where RCI refers to a Random Consistency Index. A CR of 0.1 or less is considered acceptable. If the CR is over the acceptable value, then inconsistency in pairwise comparison judgements has occurred and this process should be reviewed, reconsidered and improved.

TOPSIS is a compromise ranking method developed to determine best alternative nearest to the positive ideal solution and farthest from the negative ideal solution (Hwang, & Yoon, 1981). It has been successfully applied in many different research areas (e.g. Supply Chain Management, Manufacturing Systems, and Energy Management) (Behzadian, Kazemzadeh, Albadvi, & Aghdasi, 2012). The stepwise procedure of TOPSIS method is as follows (Hwang, & Yoon, 1981):

Step 1: Construct normalized decision matrix:

$$r_{ij}(x) = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}}, \quad i = 1, \dots, n; \quad j = 1, \dots, m \quad (19)$$

Where x_{ij} and r_{ij} are original and normalized score of decision matrix, respectively.

Step 2: Construct the weighted normalized decision matrix:

$$v_{ij}(x) = w_j * r_{ij}(x), \quad i = 1, \dots, n; \quad j = 1, \dots, m \quad (20)$$

Where w_j is the weight of the j criterion.

Step 3: Determine the positive ideal solution (PIS) and negative ideal solution (NIS):

$$\begin{aligned} PIS = A^+ &= \{v_1^+(x), v_2^+(x), \dots, v_j^+(x), \dots, v_m^+(x)\} \\ &= \left\{ \left(\max_i v_{ij}(x) \mid j \in J_1 \right), \left(\min_i v_{ij}(x) \mid j \in J_2 \right) \mid i = 1, \dots, n \right\} \end{aligned} \quad (21)$$

$$\begin{aligned} NIS = A^- &= \{v_1^-(x), v_2^-(x), \dots, v_j^-(x), \dots, v_m^-(x)\} \\ &= \left\{ \left(\min_i v_{ij}(x) \mid j \in J_1 \right), \left(\max_i v_{ij}(x) \mid j \in J_2 \right) \mid i = 1, \dots, n \right\} \end{aligned} \quad (22)$$

Where J_1 and J_2 are the benefit and the cost attributes, respectively.

Step 4: Calculate the separation values from PIS and NIS for each alternative:

$$D_i^* = \sqrt{\sum_{j=1}^m [v_{ij}(x) - v_j^+(x)]^2}, \quad i = 1, \dots, n \quad (23)$$

and

$$D_i^- = \sqrt{\sum_{j=1}^m [v_{ij}(x) - v_j^-(x)]^2}, \quad i = 1, \dots, n \quad (24)$$

Step 5: Calculate the relative closeness to the ideal solution:

$$C_i^* = D_i^- / (D_i^* + D_i^-), \quad i = 1, \dots, n \quad (25)$$

Where $C_i^* \in [0,1] \forall i = 1, \dots, n$

Finally, the alternatives can be ranked by respective C_i^* value in descending order.

The VIKOR method was introduced as a technique for multi-criteria optimization of complex systems (Opricovic, 1998). It determines the compromise ranking list and the compromise solution. Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The compromise ranking algorithm VIKOR has the following steps (Opricovic & Tzeng, 2004):

Step 1: Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \dots, n$.

If the i -th function represents a benefit, then:

$$f_i^* = \max_j f_{ij}, \quad f_i^- = \min_j f_{ij}$$

If the i th function represents a cost, then:

$$f_i^* = \min_j f_{ij}, \quad f_i^- = \max_j f_{ij}$$

Step 2: Compute the values S_j and $R_j, j = 1, 2, \dots, J$, by the relations

$$S_j = \sum_{i=1}^n w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-) \quad (26)$$

$$R_j = \max_i [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)] \quad (27)$$

where w_i are the weights of criteria, expressing their relative importance for the decision maker.

Step 3: Compute the values $Q_j, j = 1, 2, \dots, J$, by the relation

$$Q_j = \frac{v(S_j - S^*)}{(S^- - S^*)} + \frac{(1-v)(R_j - R^*)}{(R^- - R^*)} \quad (28)$$

where

$$\begin{aligned} S^* &= \min_j S_j, & S^- &= \max_j S_j \\ R^* &= \min_j R_j, & R^- &= \max_j R_j \end{aligned}$$

and v is introduced as weight of the strategy of the maximum group utility, whereas $1 - v$ is the weight of the individual regret.

Step 4: Rank the alternatives, sorting by the values S , R and Q , in decreasing order. The results are three ranking lists.

Step 5: Propose as a compromise solution the alternative (a') which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

C1: "Acceptable advantage"

$$Q(a'') - Q(a') \geq DQ$$

where a'' is the alternative in the second position in the ranking list by Q ; $DQ = 1/(J - 1)$; J is the number of alternatives.

C2: "Acceptable stability in decision making":

Alternative (a') must also be the best ranked by S or/and R .

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

Alternatives a' and a'' if only condition C2 is not satisfied, or

Alternatives $a', a'', \dots, a^{(M)}$ if condition C1 is not satisfied; and $a^{(M)}$ is determined by the relation $Q(a^{(M)}) - Q(a') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

RESULTS AND DISCUSSION

In order to analyse the performance of different MCDM methods, a real-word decision-making problem of cookies recipe selection was used. Table 1 summarizes the information which are relevant for the use of MCDM methods.

Table 1. Parameters for analysis.

	Criteria	Price	Preparation time	Calories	Review	Number of cookies	Visual appearance
Alternatives	Unit	RSD	Minutes	kCal	-	-	5-point
	Min/Max	Min	Min	Min	Max	Max	Max
	Weight	0.2	0.1	0.1	0.3	0.2	0.1
	AllRecipes	584	60	298	4,6	24	5
	Tasty	310	65	273	4,8	12	3
	BBC Good Food	285	25	121	4,8	30	7
	Joy Food Sunshine	484	30	183	5	36	5
	Betty Crocker	442	90	120	4,5	48	9

Based on the information provided in Table 1, the evaluation of the proposed alternatives has been conducted using PROMETHEE, ELECTRE, AHP, TOPSIS, and VIKOR. The ranking of the alternatives using all the aforementioned methods are presented in Table 2.

Table 2. Ranking of the alternatives.

PROMETHEE	ELECTRE	AHP	TOPSIS	VIKOR
BBC Good Food (0.4000)	BBC Good Food (2)	Joy Food Sunshine (0.2884)	BBC Good Food (0.6671)	BBC Good Food (0.0000)
Betty Crocker (0.2500)	Tasty (1)	BBC Good Food (0.1965)	Betty Crocker (0.6625)	JoyFood Sunshine (0.0879)
JoyFood Sunshine (0.0250)	JoyFood Sunshine (1)	Tasty (0.1878)	JoyFood Sunshine (0.5901)	Tasty (0.5281)
Tasty (-0.2500)	Betty Crocker (1)	Betty Crocker (0.1730)	Tasty (0.3409)	Betty Crocker (0.7325)
AllRecipes (-0.4250)	AllRecipes (0)	AllRecipes (0.1542)	AllRecipes (0.2875)	AllRecipes (0.8333)

Results presented in Table 2 are showing disagreement in the ranking of the alternatives based on the method used. Majority of methods, except AHP, are favouring BBC good Food as the most preferred alternative. Possible explanation for the difference of the most preferred alternative when applying AHP could be found in the fact that AHP is a qualitative method where the subjectivity of the decision maker affects results greatly. The only thing that is common for all the methods is the ranking of the AllRecipes at the last place. Furthermore, it is interesting to notice that only PROMETHEE and TOPSIS has the same ranking of all alternatives even though these methods do not belong to the same group of methods.

Analysing results presented in this paper, we can conclude that complex decisions which consider evaluation of a large number of alternatives evaluated based on a set of conflicting criteria are highly dependable on the method used for problem solving. This implies that different conclusions may be generated only because of a method that has been selected for solving a certain problem. It is important to emphasize that differences in the results presented by various methods do not mean that one approach should be favoured over another. It simply implies that it is important for decision makers to get familiar with different MCDM methods and their specificities so they can apply appropriate method for problem solving based on their preferences, since at the beginning of the decision-making process, decision maker is not aware of the possible outcome and therefore any result can be potentially acceptable.

CONCLUSIONS

The research presented in this paper is focused on MCDM methods and one of the important problems recognized in this field. More specifically, this paper tackles the problem of yielding different results when solving the same decision-making problem with different MCDM methods. Analysis presented in this paper show that five of the most used MCDM methods (i.e., PROMETHEE, ELECTRE, AHP, TOPSIS, VIKOR) do not produce the same ranking of alternatives when applied to the same problem. Therefore, this paper contributes to the literature in the field by confirming the raised issue and trying to explain some of the inconsistencies that were perceived.

This research is limited on observing the inconsistencies in results of MCDM methods which are the consequence of differences in algorithms of these methods. Further research should include other issues, such as changing of criteria weights or other parameters that affect results, which are relevant for the use of different MCDM methods.

ACKNOWLEDGEMENT

The results presented in this paper are part of the research within the project “Exploring the possibilities of applying artificial intelligence in industrial engineering, management and teaching process at DIEM”, Department of Industrial Engineering and Management, Faculty of Technical Sciences, University of Novi Sad, Republic of Serbia.

LITERATURE

- Anojkumar, L., Ilangkumaran, M., & Sasirekha, V. (2014). Comparative analysis of MCDM methods for pipe material selection in sugar industry. *Expert Systems with Applications*, 41(6).
- Behzadian, M., Kazemzadeh, R. B., Albadvi, A., & Aghdasi, M. (2010). Promethee: A comprehensive literature review on methodologies and applications. *European Journal of Operational Research*, 200(1).
- Behzadian, M., Khanmohammadi O. S., Yazdani, M., & Ignatius, J. (2012). A state-of the-art survey of TOPSIS applications. *Expert Systems with Applications*, 39(17), 13051–13069.
- Benayoun, R., Roy, B., & Sussman, B. (1966). Electre: Une méthode pour guider le choix en présence de points de vue multiples. *Semametra International*, 49.
- Brans, J. P. (1982). *L'ingénierie de la décision: l'élaboration d'instruments d'aide a la décision*. Université Laval, Faculté des sciences de l'administration.
- Chu, M. T., Shyu, J., Tzeng, G. H., & Khosla, R. (2007). Comparison among three analytical methods for knowledge communities group-decision analysis. *Expert Systems with Applications*, 33(4), 1011–1024.
- Guitouni, A., & Martel, J.M. (1998). Tentative guidelines to help choosing an appropriate MCDA method. *European Journal of Operational Research*, 109(2), 501–521.
- Hwang, C. L., & Yoon, K. (1981). Methods for multiple attribute decision making. In *Multiple attribute decision making* (pp. 58-191). Springer, Berlin, Heidelberg.
- MacCrimmon, K. R. (1973). An overview of multiple objective decision making. *Multiple criteria decision making*, 3, 18-44.
- Mardani, A., Jusoh, A., Nor, K., Khalifah, Z., Zakwan, N., & Valipour, A. (2015). Multiple criteria decision-making techniques and their applications—a review of the literature from 2000 to 2014. *Economic research-Ekonomska istraživanja*, 28(1), 516-571.
- Opricovic, S. (1998). Multicriteria optimization of civil engineering systems. *Faculty of Civil Engineering, Belgrade*, 2(1), 5-21.
- Opricovic, S., & Tzeng, G. H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455.
- Opricovic, S., & Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. *European Journal of Operational Research*, 178(2).
- Ozernoy, V. M. (1992). Choosing the best multiple criteria decision-making method. *Infor*, 30(2), 159–171.

- Roy, B. (1978). Electre III: un algorithme de methode de classements fonde sur une representation floue des préférences en presence de critères multiples. *Cahieres de CER*, 20(1), 3–24.
- Roy, B. ., & Bertier, P. (1973). *La méthode ELECTRE II*, 142.
- Saaty, T. L. (1980). *The Analytic Hierarchy Process*. New York, United States: McGraw-Hill.
- Vincke, J. P. (1985). A Preference Ranking Organisation Method: (The PROMETHEE Method for Multiple Criteria Decision-Making). *Management Science*, 31(6), 647–656.
- Yu, W. (1992). ELECTRE TRI (aspects méthodologiques et manuel d'utilisation). *Document-Université de Paris-Dauphine, LAMSADE*.
- Zanakis, S. H. S. H., Solomon, A., Wishart, N., & Dublisk, S. (1998). Multi-attribute decision making: A simulation comparison of select methods. *European Journal of Operational Research*, 107(3), 507–529.